Example problem: Factorial ANOVA

A researcher is interested in what affects the amount of candy that is given out during Halloween. She randomly selects 15 5-year-olds, 15 10-year-olds and 15 15-year-olds and randomly assigns them to wear either a funny, cute or scary costume. Each participant is sent to the same randomly selected house in a middle income neighborhood to trick-or-treat and the number of pieces of candy was recorded for each participant.

Type of Halloween Costume

Age of Child		Funny	Cute	Scary
	5 years old	12	11	16
	5 years old	12	8	18
	5 years old	14	8	17
	5 years old	15	9	12
	5 years old	16	9	17
	10 years old	14	6	11
	10 years old	14	2	10
	10 years old	15	7	12
	10 years old	13	7	13
	10 years old	11	5	12
	15 years old	5	4	14
	15 years old	5	6	12
	15 years old	5	3	13
	15 years old	4	1	9
	15 years old	8	5	16

Marginal Means

Funny = 10.867 Cute = 6.067 Scary = 13.467

5 = 12.933 10 = 10.133 15 = 7.333

Marginal Sums

Funny = 163 Cute = 91 Scary = 202

5 = 194 10 = 152 15 = 110

Grand Mean = 10.13333

Total = 456

$\Sigma Y^2 = 5514$

Cell Means

	Funny	Cute	Scary
5	13.8	9.0	16.0
10	13.4	5.4	11.6
15	5.4	3.8	12.8

Cell Sums

	Funny	Cute	Scary
5	69	45	80
10	67	27	58
15	27	19	64

Cell Standard Deviations

	Funny	Cute	Scary
5	1.789	1.225	2.345
10	1.517	2.074	1.140
15	1.517	1.924	2.588

On the next page, perform a Factorial ANOVA and show all 7 hypothesis testing steps.

1. **State Null Hypothesis:**

   i. $h_0 : \mu_{\text{funny}}$ ____ μ_{cute} ____ μ_{scary} (Main effect for Costume)

   ii. $h_0 : \mu_{\text{age5}}$ ____ μ_{age10} ____ μ_{age15} (Main effect for Age)

   iii. $h_0 : \mu_{\text{age5funny}}$ ____ μ_{age5cute} ____ $\mu_{\text{age5scary}}$ ____
   $\mu_{\text{age10funny}}$ ____ $\mu_{\text{age10cute}}$ ____ $\mu_{\text{age15scary}}$
   $\mu_{\text{age15funny}}$ ____ $\mu_{\text{age15cute}}$ ____ $\mu_{\text{age15scary}}$ (Interaction Effect)

2. **Alternative Hypothesis:**

   i. $h_1 : \textit{At least } 2 \text{ costume } \mu s \textit{ are different}$

   ii. $h_1 : \textit{At least } 2 \text{ Age } \mu s \textit{ are different}$

   iii. $h_1 : \textit{At least } 2 \text{ Cell } \mu s \textit{ are different}$

3. **Decide on α (usually .05):** α = ____

4. **Decide on type of test (distribution; *z*, *t*, F, etc.)**
   Questions to ask:
   a. How many groups do you have?
      i. Only 2 than you can use a t-test.
      ii. Are the groups arranged within multiple independent variables?
      iii. More than 2 groups/IVs → ANOVA → F distribution
   b. Can we treat the scores as independent (e.g. they are NOT from the same person, they are NOT matched subjects, they are NOT related subjects, etc.)?
      If Yes, then continue with the between groups ANOVA
      If No, STOP you may need to perform a repeated measures ANOVA
   c. Can we assume a normally distributed sampling distribution?
      In other words, do we have 20+ degrees of freedom for the WG source of variance?
      If yes, then continue.
      If no, do not continue the test cannot be performed.
   d. Do the groups have homogenous variances?

   $F_{MAX} = \frac{s^2_{\text{Largest}}}{s^2_{\text{Smallest}}} = \frac{\_\_\_\_}{\_\_\_\_} =$ _______ , if this value is smaller than 3 proceed.

5. **Find critical value & state decision rule(s)**
   a. For F_{cv} you need both df_{Effect} = #groups$_{\text{(effect)}}$ – 1 and df_{WG} = *N - #cells*. Table D.3 F_{cv} (df_{effect}, df_{WG}), if $F_o > F_{cv}$ reject the null hypothesis
   b. So there are 3 costume groups therefore $df_{Costume}$ = ____ – 1 = ____ and df_{WG} = ____ - 1 = 36. Table D.3 F_{cv} (___,___) = ______, if F_o > _______ reject the null hypothesis

c. So there are 3 age groups therefore df_{Age} = _____ – 1 = _____ and df_{WG} = _____ - 1 = 36. Table D.3 F_{cv} (____,____) = _______, if F_o > ________ reject the null hypothesis
d. So there are 9 cells for the interaction therefore $df_{Interaction(Costume*Age)}$ = _____ – 1 = _____ and df_{WG} = _____ - 1 = 36. Table D.3 F_{cv} (____,____) = _______, if F_o > ________ reject the null hypothesis

6. **Calculate test**
   - You can use the Deviation Approach but it will take you a long time, so for the sake of our collective sanities let's just focus on the computational approach (but the deviation approach should achieve the same results

$$SS_{\text{costume}} = \frac{\sum\left(\sum A\right)^2}{bn} - \frac{T^2}{abn} = \frac{163^2 + \_\_\_^2 + \_\_\_^2}{(\_\_)(\_\_)} - \frac{(\_\_\_)^2}{(\_\_)(\_\_)(\_\_)} = \frac{\_\_\_\_\_}{\_\_\_} - \frac{\_\_\_\_\_}{\_\_\_} =$$

$$= \_\_\_\_\_\_ - \_\_\_\_\_\_ = \_\_\_\_\_\_$$

$$SS_{Age} = \frac{\sum\left(\sum B\right)^2}{an} - \frac{T^2}{abn} = \frac{\_\_\_^2 + \_\_\_^2 + 110^2}{(\_\_)(\_\_)} - 4620.8 = \frac{\_\_\_\_\_}{\_\_\_} - 4620.8 =$$

$$= \_\_\_\_\_\_ - 4620.8 = \_\_\_\_\_\_$$

$$SS_{AB} = \frac{\sum\left(\sum AB\right)^2}{n} - \frac{\sum\left(\sum A\right)^2}{bn} - \frac{\sum\left(\sum B\right)^2}{an} + \frac{T^2}{abn} =$$

$$\frac{69^2 + 45^2 + \_\_\_^2 + \_\_\_^2 + \_\_\_^2 + \_\_\_^2 + \_\_\_^2 + \_\_\_^2 + \_\_\_^2}{\_\_} - 5043.6 - 4856 + 4620.8 =$$

$$\frac{\_\_\_\_\_}{\_\_} - 5043.6 - 4856 + 4620.8 = \_\_\_\_\_\_ - 5043.6 - 4856 + 4620.8 = 112$$

$$SS_{S/AB} = \sum Y^2 - \frac{\sum\left(\sum AB\right)^2}{n} = 5514 - 5390.8 = \_\_\_\_\_$$

$$SS_{Total} = \sum Y^2 - \frac{T^2}{abn} = 5514 - 4620.8 = \_\_\_\_\_$$

Source	SS	df	MS	F
Costume	_____	__	_____	_____
Age	_____	__	_____	_____
AB	112	__	28	_____
S/AB	_____	36	_____	
Total	_____			

7. **Apply Decision Rule(s)**
   - **Costume**: Since, ________ (i.e. observed value) _____ (i.e. >, <) ________ (critical value), __________(i.e. **DO or DO NOT)** reject the null hypothesis.
   - **Age**: Since, ________ (i.e. observed value) _____ (i.e. >, <) ________ (critical value), __________(i.e. **DO or DO NOT)** reject the null hypothesis.
   - **Interaction**: Since, ________ (i.e. observed value) _____ (i.e. >, <) ________ (critical value), __________(i.e. **DO or DO NOT)** reject the null hypothesis.